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NUMERICAL MODELING OF IDEAL INCOMPRESSIBLE FLUID FLOW OVER A STEP

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Results of calculations of fluid flow over a step located on a channel bottom are given. Numerical modeling is performed for the model of free-boundary potential flows of an ideal incompressible fluid using a finite-difference method with dynamically adaptive grids. The behavior of the free surface in the neighborhood of the step is studied as a function of the incident-flow velocity. The results are compared with experimental data.

Key words: *ideal fluid, free boundary, step at the bottom, adaptive grids, wave profiles.*

Introduction. The interaction of a steady-state fluid flow with a step at the bottom of a channel has been studied experimentally [1]. It has been shown that even for subcritical fluid velocities above the step, there may be the formation of a wavy free boundary of the type of a steady-state undular bore that arises in flow over a rectangular sill [2]. The occurrence of such a wave-like stationary pattern for some relations between the step height and the fluid discharge is attributed [1] to the substantial non-one-dimensional nature of the flow in the neighborhood of the step and the flow separation observed there. In addition, it is noted [1] that even in the case of a simple obstacle such as a step, the flow contains subcritical and supercritical regions, stagnant zones, and complex vortex structures. The flow is affected by wall friction. Thus, for a detailed description of the flow pattern, it is necessary to use a mathematical model that takes into account the above-mentioned factors. It should be noted that for obstacles with a smooth contour, some flow characteristics are adequately predicted by approximate models. Thus, Liapidevskii and Teshukov [3] give improved shallow-water models that accurately describe transcritical flow over obstacles with a smooth profile, including flows with the formation of hydraulic jumps of the undular bore type. For obstacles of nonsmooth shape, for example, a step, calculations for the shallow-water model with a detailed description of the flow give considerable errors because the flow pattern near such obstacles is mainly affected by vertical displacements of the fluid.

In the present work, the flow over a step is studied using a finite-difference algorithm based on the model of plane potential free-boundary flows of an ideal incompressible fluid. This model is also approximate since it was obtained ignoring many details of real flows but taking into account the variation in the fluid parameters with depth. Therefore, in the neighborhood of the obstacle, where vertical accelerations of the fluid are comparable to or even exceed horizontal accelerations, it is possible to obtain a more accurate pattern of flow interaction with the obstacles than that using classical shallow-water models. The computational algorithm is described in detail in [4]. The same paper gives results from calculations of the dependence of the difference between the fluid surface levels ahead of and behind the step on the incident-flow velocity. The present work, which is a continuation of the studies started in [4], is devoted mainly to the behavior of the free boundary of steady-state flow in the neighborhood of a step.

1. Solution Technique. We consider plane–parallel flow of a fluid layer with surface waves. From below, the layer is bounded by a horizontal bottom with is a step of height b and infinite length (Fig. 1). From above, the fluid is bounded by a free boundary. The fluid flow of known velocity $u_{-\infty}$ which is uniform at infinity is

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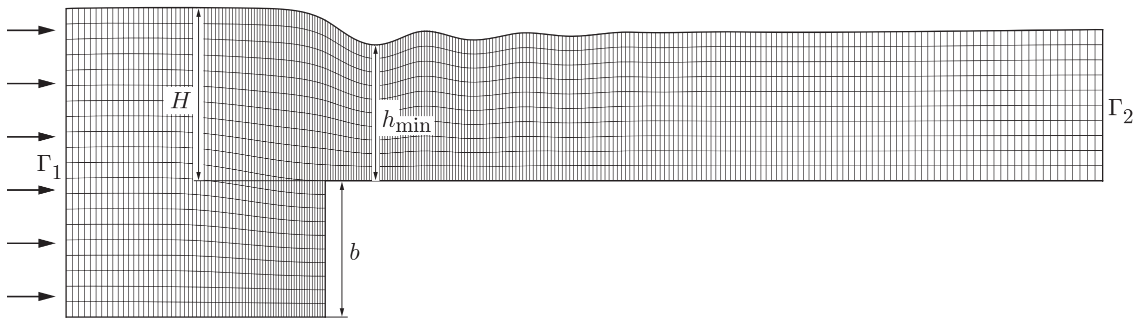

Fig. 1. Computational grid and notation.

assumed to be incident on the step from the left. We consider the fluid layer bounded on the left and right by the segments Γ_1 and Γ_2 of the vertical straight lines $x = 0$ and $x = L$, respectively (x is the horizontal axis). On the boundary Γ_1, a uniform flow of velocity $u_{-\infty}$ is specified, and on the boundary Γ_2, a nonreflecting boundary condition [4] is imposed, which provides for free passage of waves through this boundary.

The mathematical formulation of the problem for potential flows consists of determining the velocity potential satisfying the Laplace equation and the function $y = \eta(x, t)$ (y is the vertical coordinate and t is time) describing the free boundary, on which kinematic and dynamic conditions should be satisfied.

In the calculations, we used grids movable in the vertical direction and adjustable to the flow boundaries. In the horizontal direction, the grid was made significantly finer in the neighborhood of the step, and away from it, the grid steps increased in a geometric progression (Fig. 1).

The steady-state flow over the step was calculated by a relaxation method. At the initial time $t = 0$, we specified a unperturbed free boundary $\eta \equiv 0$ and determined the velocity components corresponding to the fluid flow in a region with impenetrable upper and lower walls and with a specified uniform inflow through the boundary Γ_1 and a uniform discharge through the boundary Γ_2. Thus, at $t = 0$, steady-state flow in the channel with a solid upper wall $y = 0$ was specified. The bottom depth ahead of the step was set equal to h_0. At $t > 0$, the upper wall was removed, and a substantially unsteady flow with a free boundary arose, which subsequently became steady-state. In each time layer, the new values of the potential on the free boundary were calculated, the potential inside the region was found, the new position of the free boundary for the given time layer was determined, and a grid for the next time layer was constructed.

There are some differences between the formulation of the boundary conditions considered and the experimental conditions of [1]. First, in the present study, the inflow velocity rather than the fluid discharge rate was specified in the entrance region. The fluid pressure head and, hence, the discharge at the entrance were determined during the establishment of the flow. Second, the conditions at the exit were different. In the experiments of [1], water freely discharged into the atmosphere, whereas in the present calculations, a nonreflecting boundary condition was imposed at the exit. This difference resulted in a discrepancy between calculated free-boundary profiles and experimental profiles near the exit.

2. Results of Numerical Modeling. Numerical experiments were performed for a step separated by a distance $x/h_0 = 15$ from the boundary Γ_1. The boundary Γ_2 was at a distance $x/h_0 = 120$.

Figure 2 shows free-boundary profiles for steady-state flow for $b/h_0 = 0.33$ and various values of the incident-flow velocity $U_- = u_{-\infty}/\sqrt{gh_0}$. As in the experiments of [1], at low incident-flow velocities, only a small reduction in the level near the step is observed (curves 1 and 2). As the velocity $u_{-\infty}$ increases, the stationary free boundary becomes wavy and the wavelength also increases with increasing $u_{-\infty}$ (curves 5–10), which agrees with the results of the experiments. Finally, beginning from a certain value of $u_{-\infty}$, the waves on the fluid surface disappear, and away from the sharp edge of the step, the free boundary takes the shape of a rounded ledge with a constant level.

In [1], the existence of waves for steady-state flow over a step is explained by flow separation from the step. Of course, the potential ideal flow model used in the present calculations cannot reproduce the separation; nevertheless, waves on the fluid surface are present in a certain range of the velocity $u_{-\infty}$. Figure 3 shows the velocity field in the immediate proximity of a step of height $b/h_0 = 0.33$ for $u_{-\infty}/\sqrt{gh_0} = 0.43$ (the free boundary is shown by curve 7 in Fig. 2). It is evident that in the numerical experiment, the fluid rounds the edge of the step

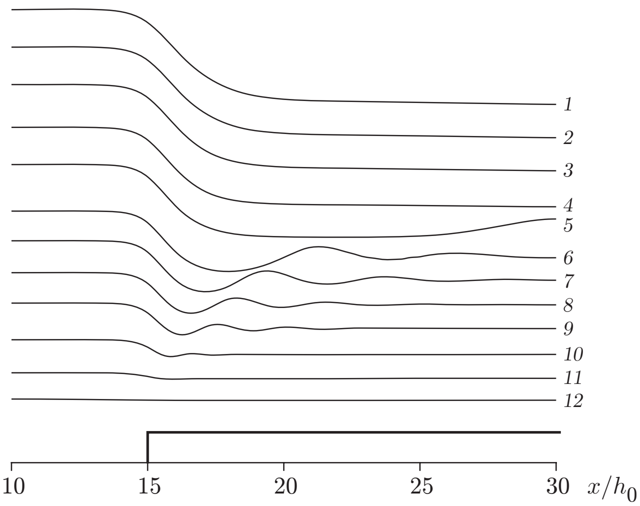

Fig. 2. Shape of the free boundary at $b/h_0 = 0.33$ and incident-flow velocities $U_=0.63$ (1), 0.6 (2), 0.57 (3), 0.53 (4), 0.5 (5), 0.45 (6), 0.43 (7), 0.4 (8), 0.37 (9), 0.3 (10), 0.2 (11), and 0.1 (12).

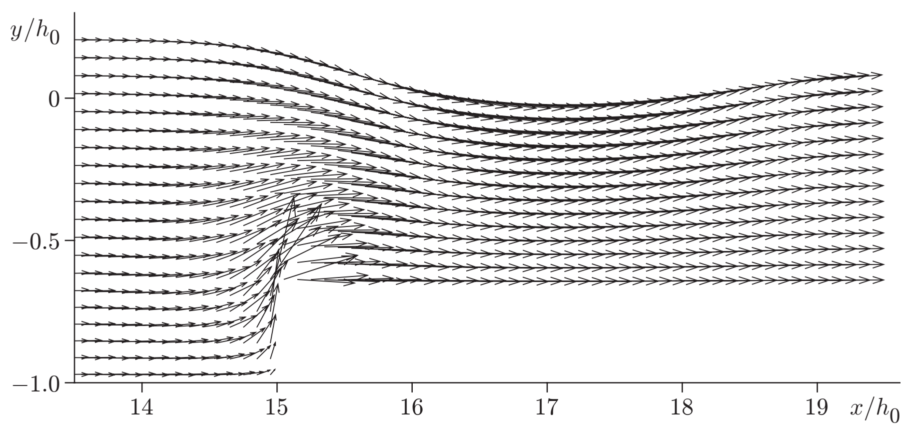

Fig. 3. Velocity field in the neighborhood of the step ($b/h_0 = 0.33$ and $U_- = 0.43$).

without separation but at a very high velocity, whose vector ahead of the edge is directed almost perpendicularly to the main flow. In this case, for moderate values of $u_{-\infty}$, the flow zone perturbed by the edge has a greater extent in the vertical direction than the total fluid depth. For small or very large values of $u_{-\infty}$, the relative dimension of this zone is insignificant and this zone has a weak influence on the flow over the step; this is probably the reason why the free boundary calculated for the potential flow model is not wavy in these cases.

Figure 4 shows the effect of the incident-flow velocity on the pressure head at the entrance section for various values of the step height. It is evident that the pressure head increases monotonically with increasing values of $u_{-\infty}$. For small values of $u_{-\infty}$, the dependence $H(u_{-\infty})$ is nearly quadratic, and for large values of $u_{-\infty}$, it is nearly linear.

The calculated values of the pressure head H were used to determine the critical fluid depth behind a step in a channel with a horizontal bottom by the formula

$$\frac{h_*}{h_0} = \Big(\frac{H+b}{h_0}\,U_-\Big)^{2/3}. \tag{1}$$

The corresponding dependences are shown by curves 4–6 in Fig. 5. Curves 1–3 show the dependence of the minimum depth $h_{\min}$ on $u_{-\infty}$. If the fluid surface was not wavy, the value of $h_{\min}$ was set equal to the constant fluid level that established away from the edge. It is evident that for each height of the step there is a certain threshold value of the velocity $u^*_{-\infty}(b)$. For $u_{-\infty} < u^*_{-\infty}(b)$, the value of $h_{\min}$ depends weakly on $u_{-\infty}$, and for $u_{-\infty} > u^*_{-\infty}(b)$, the

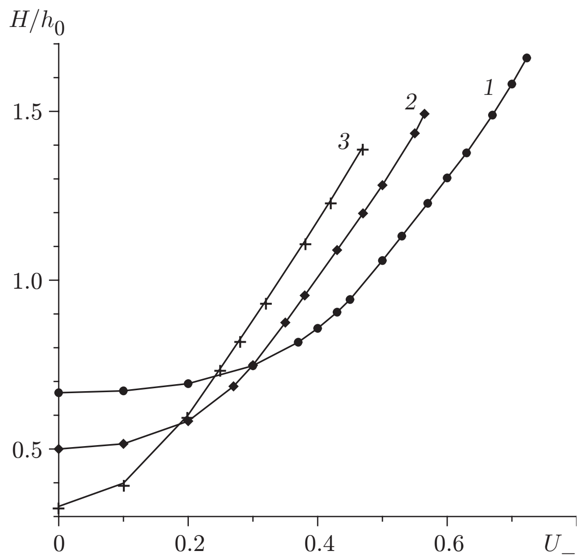

Fig. 4. Dependence of the pressure head on the incident-flow velocity for various values of the step height: $b/h_0 = 0.33$ (1), 0.5 (2), and 0.67 (3).

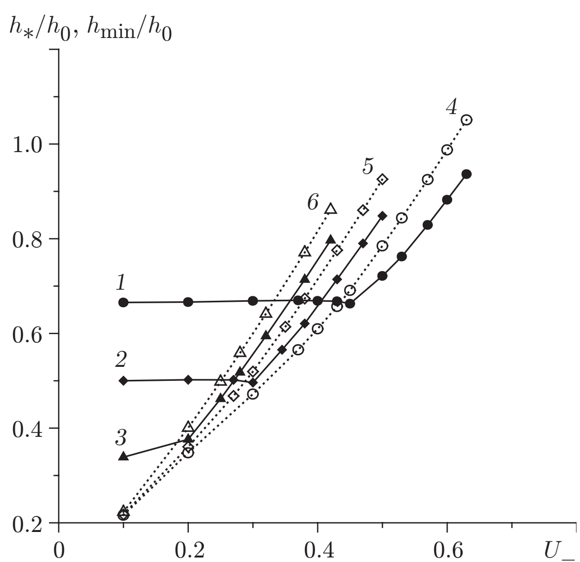

Fig. 5. Minimum depth $h_{\min}/h_0$ above the step (1–3) and critical depth h_*/h_0 (4–6) versus incident-flow velocity for various values of the step height: $b/h_0 = 0.33$ (1 and 4), 0.5 (2 and 5), and 0.67 (3 and 6).

value of $h_{\min}$ depends significantly on $u_{-\infty}$ and this dependence is similar to the dependence $h_*(u_{-\infty})$. In addition, as shown in Fig. 5, in the neighborhood of the point $u^*_{-\infty}(b)$, transition from subcritical to supercritical flow occurs at a Froude number somewhat larger than unity (the critical depth becomes greater than the minimum value). Moreover, in the neighborhood of the point $u^*_{-\infty}(b)$, there is a transition from the wavy free boundary (curve 6 in Fig. 2) to a boundary without waves (curve 4).

In Fig. 6, the points show the free-boundary profile obtained in the experiment of [1] for $h_* = 7.45$ cm and $b = 4.85$ cm, which for $h_0 = 14.55$ cm corresponds to the dimensionless value of $b/h_0 = 0.33$. For a quantitative comparison of the numerical modeling results with experimental data, we performed a calculation of the flow over a step of the indicated height for an incident-flow velocity $U_- = 0.31$ calculated by formula (1) using the value of

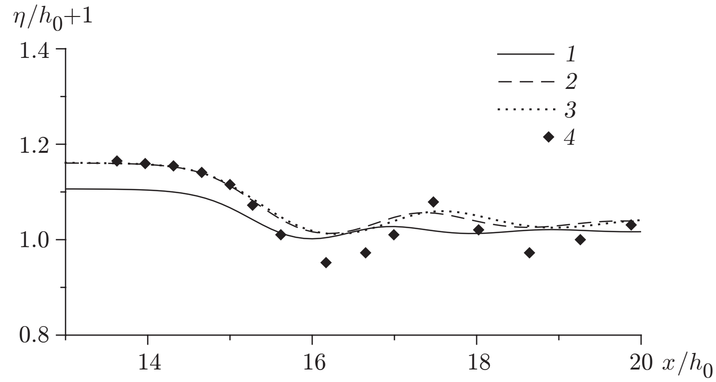

Fig. 6. Steady-state free surface profile for $b/h_0 = 0.33$ and $U_- = 0.31$ (curve 1 and points 4), $b/h_0 = 0.39$ and $U_- = 0.31$ (curve 2), and $b/h_0 = 0.33$ and $U_- = 0.38$ (curve 3); curves 1–3 refer to the present calculations and points 4 to the experiment of [1].

$H + b = 16.95$ cm taken from [1]. In Fig. 6, it is evident that for the same initial data, the calculated free-boundary profile ahead of the step (curve 1) is below the experimental profile. Accordingly, the discharge is also smaller. In addition, the wavy pattern of the flow is less pronounced in the calculations. It should be noted that in real flows there is flow separation from the frontal edge of the step and, in fact, not only the step but also the separation zone above it are in the flow. Therefore, it can be assumed that calculations using the ideal fluid model will provide the best agreement with the experimental data for a step whose height is increased by the thickness of the separation zone compared to the specified height, or for the specified step at a higher approach velocity. The nature of curves 2 and 3 in Fig. 6 confirms this assumption. It is evident that ahead of the step, the free-boundary profiles obtained numerically and experimentally coincide. However, in this case, too, the waves behind the step are less intense in the calculations than in the experiments.

The results presented here suggest that even a simplified model such as the potential ideal flow model can be used to predict the fluid flow pattern over a step because it provides satisfactory agreement between computational and the experimental data.

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